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The sharp peak-flat trough pattern and critical speculation

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Abstract. We find empirically a characteristic sharp peak-flat trough pattern in a large set of commodity prices. We argue that the sharp peak structure reflects an endogenous inter-market organization, and that peaks may be seen as local "singularities" resulting from imitation and herding. These findings impose a novel stringent constraint on the construction of models.

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1 Introduction

In the stock market, returns over long period of times are often mainly due to rare large upward price variations that occur over a tiny fraction of the total trading time: the US equity index $S\&P500$ for instance has gained an average of 16% a year from 1983 to 1992 and 80% of this return stems from forty days of trading, *i.e.* less than 1.6% of all working days. This property is shared by other markets and other assets. The prices of commodities that we will investigate here is no exception.

The prices of most commodities are also characterized by rare and sudden bursts of apparently outlying values. A typical and spectacular example is provided by the evolution of the price of gold and silver; in the half century since 1950, these prices experienced one huge peak that lasted for two decades and resulted in a multiplication of the (deflated) price by a factor of the order of 10.

These two examples outline the importance of both rare events and the effect of correlations in the determination of market price time series. The Gaussian paradigm of independent normally distributed price increments [3,48] has long been known to be incorrect with many attempts to improve it. Econometric nonlinear autoregressive models with conditional heteroskedasticity (ARCH) [17] and their generalizations [6] as well as jump-diffusion models [29] capture only imperfectly the volatility (variance) correlations and the fat tails of the probability density distribution (pdf) of price variations. Alternatively, the fat tail properties of the full pdf (corresponding to a one-point statistics) has been described by a Lévy law [34] for cotton and other commodities and more recently by a truncated Lévy flight [1,37] for equities or by a superposition of Gaussian pdf's with log-normally distributed variance [21]. A recent decomposition of the volatility (standard deviation) of return data across scales of several financial time series has revealed the existence of a causal information cascade from large scales to fine scales that expresses itself in the volatilities [2].

In very liquid markets of equities and foreign exchanges for instance, correlations of price variations are extremely small, as any significant correlation would lead to an arbitrage opportunity that is rapidly exploited and thus washed out. Indeed, the fact that there are almost no correlations between price variations in liquid markets can be understood from the following simple calculation presented in [8] that we slightly extend. For the sake of illustration, let us assume that the price variations δx over the time interval δt are Gaussian correlated stationary variables of zero average. Time is expressed as a multiple of δt. The time series $\delta x_1, \delta x_2, ..., \delta x_t$ is denoted by the column vector X_t. The correlation function is defined by

$$\langle \delta x_n \delta x_k \rangle \equiv C(n,k), \tag{1}$$

which is $C = \langle X X^T \rangle$ in matricial notation. If the correlation functions are the only constraints, then it is straightforward to show, under the assumption of a maximum entropy principle, that the knowledge of the correlation functions is fully embedded in that of the following multivariable probability distribution for the variables δx_n

$$P(X_t) = P_0 \exp\left(-\frac{1}{2} X_t^T C^{-1} X_t\right) . \tag{2}$$

where X_t^T denotes the transpose of X, *i.e.* the uni-row matrix whose nth row is δx_n. C^{-1} is the inverse of the

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correlation matrix. Expression (2) can be rewritten explicitely as

$$P(X) = P_0 \exp\left(-\frac{1}{2} \sum_{n,m+1}^{t} C^{-1}(n,m) X_n X_m \right). \quad (3)$$

This expression shows that, conditioned to the past values of the variations $\delta x_0, ... \delta x_{t-1}$, the probability density function of δx_t is then given by

$$P(\delta x_t) = P_0 \exp\left(-\frac{C^{-1}(t,t)}{2} \left[\delta x_t - m_t \right]^2 \right), \quad (4)$$

where P_0 is a normalization factor. The average m_t of δx_t conditioned to the past is

$$m_t \equiv \frac{\sum_{i=0}^{t-1} C^{-1}(i,t) \delta x_i}{C^{-1}(t,t)}. \quad (5)$$

It may be non-zero due to the presence of correlations. A simple trading strategy consists in buying a unit stock if $m_t > 0$ (expected future price increase) and selling if $m_t < 0$ (expected future price decrease). The average gain is then $\langle |m_t| \rangle > 0$. Let us consider the short range limit where only $C^{-1}(t,t)$ and $C^{-1}(t,t-1)$ are non-zero and δt is then equal to the correlation time which is typically 5 minutes for liquid markets. The average return over one correlation time is then $\frac{1}{e} \left\langle \frac{|\delta x|}{x} \right\rangle \approx 3.7 \times 10^{-4}$ for $\left\langle \frac{|\delta x|}{x} \right\rangle \approx 10^{-3}$. Over a day, this gives an average gain of 0.59% which accrues to 435% per year when return is reinvested or 150% without reinvestment! Such small correlations would lead to substantial profits if transaction costs and other friction phenomena like slippage did not exist[1].

Counting the transaction costs of about $c \approx 0.1\%$ and since one must on average modify the position once in a correlation time to achieve the above performance, this leads to an effective average gain per transaction equal to $e^{-1} \left\langle \frac{|\delta x|}{x} \right\rangle - c$ which becomes negative in our numerical application! Since the scaling law $|\delta x| \approx \left(\frac{\delta t}{T} \right)^H$ (with $H \approx 0.6$) [41], holds for time scales less than about a day, a given level of transaction cost c allows correlations to develop over a maximum time $\delta t \approx T(ec)^{\frac{1}{H}}$ (such that $e^{-1} \left\langle \frac{|\delta x|}{x} \right\rangle = c$) without allowing arbitrage opportunities. The small level of transaction costs in efficient modern markets thus explains the low level of correlations of price variations, completing the proof of the above assertion.

The liquidity and efficiency (level of transaction costs, slippage) of markets thus control the degree of correlation that is compatible with the almost absence of arbitrage opportunity. In other words, if one detects the possibility to make money in a given market, the difficulty for entering the market and the costs limit the concrete realization of this opportunity. Less liquid markets thus allow the appearance of stronger correlations that may take more intricate forms. Here, we point out the existence of a more subtle kind of price correlations in commodities, namely the repeated existence of "sharp peak-flat trough" (SP-FT) patterns, which we will study. Identifying patterns in economic and financial prices has a long history and is often refered to as "technical analysis". Technical analysis in finance can be broadly defined as the study of financial markets, mainly using graphs of stock prices as a function of time, in the goal of predicting future trends [44]. A lot of efforts has been developed both by academic and trading institutions and more recently by physicists (using some of their statistical tools developed to deal with complex times series) to analyse past data to get informations on the future. We notice that the Dow Jones US index was also characterized by a dominance of SP-FT from 1875 to 1935, but since 1950, the sharp peaks have left place to smoother structures. This may be seen as the consequence of a more efficient arbitrage progressively destroying those patterns that would provide arbitrage opportunities.

Our motivation for this study is to provide a novel characterization, with as few fitting parameters as possible, of economic time series which can be useful for sparse data. This is a useful practical alternative (and/or complement) to the determination of statistical distributions and correlations of price changes. The problem is that in order to obtain a probability distribution function with reasonable precision, especially in the range of large price changes, huge records of several thousand prices are required. In contrast, the shape of the peaks can be analysed on fairly small samples. In restricting ourselves to the structure of the peaks, we aim at a more modest goal which is to focus on a simple sub-structure of the whole time series. The justification of this approach is simply that all scientific endeavors (may) succeed when restricting their aim to simple enough problems which, when understood, can be extended and generalized towards the more complex problems.

The main message of this paper is to point out that the shape of major price peaks of commodities varies within rather narrow limits. More specifically, the peaks are of what we call the sharp peak type: if p denotes the price, $x = \ln(p)$ turns its concavity upwards both before and after the turning point; mathematically this means that the (suitably coarse-grained) second derivative of $x(t)$ with respect to time is positive close to the peak turning point. When looking at a time series as a whole, this structure leads to the appearance of patterns that can be described as SP-FT. As a consequence, the graph of $\ln p(t)$ is *not* bottom-top symmetric: if the graph is rotated by 180° or inverted upside-down, the resulting figure will no longer resemble the graph of a genuine commodity price series. Qualitatively, this observation has been made some time ago [12]. Our purpose here is to characterize it

[1] Slippage refers to the fact that market orders are not always executed at the order price due to limited liquidity and finite human execution time.

quantitatively and propose a theoretical basis for it. It is important to realize that the existence of these SP-FT patterns put strong constraints on any general theory commodity prices. Previous attempts [11] have already highlighted the difficulty of generating peaks having realistic amplitude and frequency. If a general dynamical model for the price behavior of commodities would be available, the pattern of the peaks could be derived as a simple consequence. Some models in this spirit have indeed been investigated by a number of authors [31,32,58], but such a global approach turns out to be extremely difficult.

The shape constraint that we describe in this paper is very demanding. Thus, in order for a model to qualify, not only should it be compatible with the one-point (probability distribution) and two-points statistics (correlation functions of the volatilities), it should also correctly account for the SP-FT patterns. It is indeed important to constraint as much as possible the model construction as it has been found in the past for instance that several models could account equally well for the probability distribution [1,2,21,37]. We do not claim that the SP-FT patterns always exist for all commodities but when there are present they should be taken into account.

The second important message of the paper is that the sharp peak structure may reflect an endogenous intermarket organization, and that peaks may be seen as local "singularities". This conclusion extends and generalizes the view that the largest possible peaks in the stock market, preceeding the large crashes in the stock market, can be modelled as special critical crises [18,19,22,27,28, 52,53,57]: the underlying hypothesis is that stock market crashes may be caused by the slow buildup of powerful subterranean forces that come together in one critical instant. The use of the word "critical" is not purely literary here: in mathematical terms, complex dynamical systems such as the stock market can go through so-called "critical" points, defined as the explosion to infinity of a normally well-behaved quantity.

The paper is organized as follows. In Section 2, we describe the SP-FT patterns for the case of pre-twentieth century wheat markets. These markets have the advantage of displaying a large number of price peaks. We are thus in an ideal position for a systematic quantitative analysis. In Section 3, we consider some twentieth century commodity markets. The same pattern will be seen to hold in a number of important instances. In addition, we examine some speculative price bubbles outside the sphere of commodity markets in order to emphasize both the similarities and the discrepancies. In Section 4, we discuss some requirements to be fulfilled by a future theory and outline such a theory.

2 The sharp peak-flat trough pattern in the case of wheat markets

2.1 Wheat markets

Before 1850, wheat was the key product in the economies of Western Europe. Its production was the major task of the agricultural sector which employed over two thirds of the total manpower; its consumption was a crucial element in the diet of the masses; and finally its trade represented a large part of international trade. Because the price of wheat was of crucial importance both for the economy and for the public welfare, it was carefully recorded in every state. Some of these records extend uninterrupted from the 15th to the 20th century, and they contain many huge peaks.

A superficial view would ascribe such peaks solely to the occurrence of defavourable meteorological conditions. In fact, meteorological hazards were only the triggering factors in a more complex process. As will be seen subsequently, the average duration of the price peaks is of the order of four years, a feature that clearly speaks against the meteorological explanation since poor weather conditions during four years in a row are rather unlikely. As a matter of fact, the crucial role was played by speculation and monopolistic practices as has been convincingly documented by many historians [5,40,42,56]. Indeed, hoarding was common practice, not only by traders and retailers but also by the consumers as is shown by the following excerpt from Biollay [5]: "A survey conducted by the French State Council described how people bought up the wheat market at the first signs of a coming wave of high prices, and how they stored large amounts of wheat for future use when prices would have doubled or tripled". In short, the mechanisms by which the pre-twentieth century wheat markets operated were basically the same as those at work in modern commodity markets. One should notice an important difference, namely no futures markets existed. But some practices, such as buying the wheat in spring (*i.e.* several months before the harvest), already announced the mechanism of the forward markets. Because for wheat markets, there are so many long and reliable price records, we are in an ideal position to carry out a statistical analysis of price peaks. Such an analysis involves the following steps:

1. the selection of the peaks;
2. the definition of the parameters by which such peaks can be described;
3. the statistical analysis of those parameters.

Let us turn to the discussion of these points.

2.2 Statistical procedure

2.2.1 Selecting the peaks

Everybody would certainly agree that a peak pattern can be described as a price path which first goes up, reaches a maximum, then goes down. However, real price trajectories present in addition many short term price fluctuations; this is illustrated in Figures 1a, b, c. Figure 1c shows what the eye can identify as a large isolated peak. However, its unambiguous definition and the determination of the starting point of the raising pattern depends on the time scale at which the data is coarse-grained. If we subject the price series to a moving average procedure, local fluctuations below this time scale will be smoothed

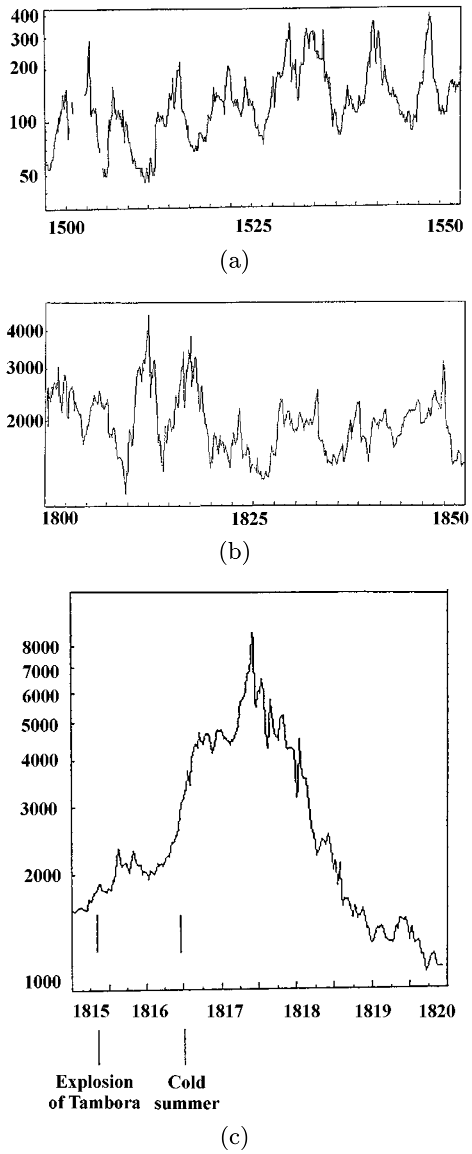


Fig. 1. (a) Monthly wheat prices in Toulouse (France) from 1500 to 1550, (b) from 1800 to 1850, (c) Weakly wheat prices in Munich during the 1815-1819 peak.

out. The width of the moving average window thus determines the desired time scale. In the present paper, we deal exclusively with monthly prices. We found that using a 41-months window smooths out adequately the local irregularities so as to get a quantitative definition that parallels the intuitive and efficient pattern recognition efficiency of the eye. We thus could eliminate the small scale structures without affecting the overall shape of the large peaks. Varying the size of the window by about 20% does not modify our results.

Let us summarize the procedure by which peaks have been systematically selected:

1. We have performed a moving average with binomial weighting coefficients and a window-width of 41 months. The resulting series will be denoted by $p_m(t)$.

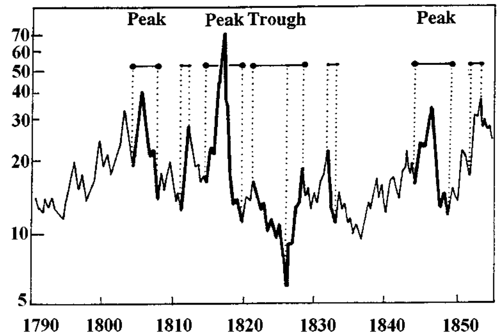


Fig. 2. Peaks and troughs of amplitude over 100% for wheat prices in Munich.

2. We then computed the first differences $d_m(t)$ of the previous series. All successive times t_i for which $d_m(t_i)$ is stricly positive were considered as belonging to the same rising path; decreasing paths were handled similarly.
3. We then computed the amplitude, $h = p_{max}/p_{min}$ of the peak from the original series $p(t)$ and we discarded the peaks for which h was less than some critical value h_c. In the following h_c will be given the two values 1.5 and 2.0.

This procedure is summarized in Figure 2 which provides the outcome of the peak identification algorithm on a specific time series. The peaks and troughs have been selected with the threshold $h_c = 2$ and are indicated by heavy dots. In addition to peaks and troughs, one could also consider isolated rising or falling paths; these have been analysed in [46]. In the present paper, we restrict ourselves to peaks and troughs.

2.2.2 Definition of the variables describing the shape of the peaks and troughs

Two sets of variables will be considered in order to describe the shape of the peaks. The first set summarizes the magnitude and symmetry of the peaks, the second set of parameters defines their shape more precisely.

In the first set, we consider five parameters defined as follows; we shall here state the definitions for the case of peaks but they can be easily deduced for the case of troughs.

1. The amplitude $h = p_{max}/p_{min}$ that has already been defined.
2. The total duration d of the peak.
3. The ratio r_p between the two prices at the beginning and at the end of the peak: r_p = initial price/final price.
4. The ratio r_d between the duration of the rising path and the duration of the falling path: r_d = duration of the path before the turning point/duration of the path after the turning point.

5. The ratio r_s between the slopes of the rising path and falling path. r_s = slope of the left hand section of the peak/slope of the right hand section.

The parameters r_p, r_d and r_s quantify the degree of (as)symmetry between the rising and falling parts of a peak. The introduction of the parameter r_s is more specifically motivated by the well-known result that Gaussian linear processes cannot rise to their maxima and fall away at different rates [9].

All parameters should be invariant under rescaling, *i.e.* when all prices are multiplied by a common factor. Otherwise those parameters would be affected by the currency in which prices are expressed. The above parameters obviously satisfy this requirement. The Figures 1c, 3 and 4 show that even the logarithms of the prices exhibit a marked concavity. The goal of the second set of parameters is to provide a measure of this concavity. For this purpose, we propose the following representation:

$$p(t) = A \exp\left[-\mathrm{sgn}(\tau)\left|\frac{t-t_0}{\tau}\right|^{\alpha}\right], \qquad (6)$$

where t_0 denotes the turning point of the peak.

- If α is equal to 1, one retrieves an exponential growth up to the turning point followed by an exponential decay. $x = \ln(p)$ is thus linear by part with a tent-like structure.
- If $\alpha < 1$ and $\tau > 0$, the function describes a sharp peak of the kind represented in Figure 1c.
- If $\alpha > 1$ and $\tau < 0$, the function describes a flat trough.
- If $\alpha > 1$ and $\tau > 0$, the function describes a "flat peak" of a kind that will be seen to exist in the real estate market.
- If $\alpha < 1$ and $\tau < 0$, the function describes a sharp trough, a rare but not altogether inexisting phenomenon.

These cases are summarized in Figure 3a. The parametrization (6) is parsimonious and intuitive. The coefficient A is just a scaling factor that depends on the currency used for expressing the price. t_0 is the date of the peak/trough, $|\tau|$ its duration and α quantifies the abruptness of the peak/trough. Close to t_0 (*i.e.* for $|t - t_0| \ll \tau$), the expression (6) can be expanded in

$$p(t)/A = 1 - \mathrm{sgn}(\tau)\left|\frac{t-t_0}{\tau}\right|^{\alpha}, \qquad (7)$$

showing a power law behavior.

2.3 Empirical results

Table 1a lists the series for which we performed a statistical analysis.

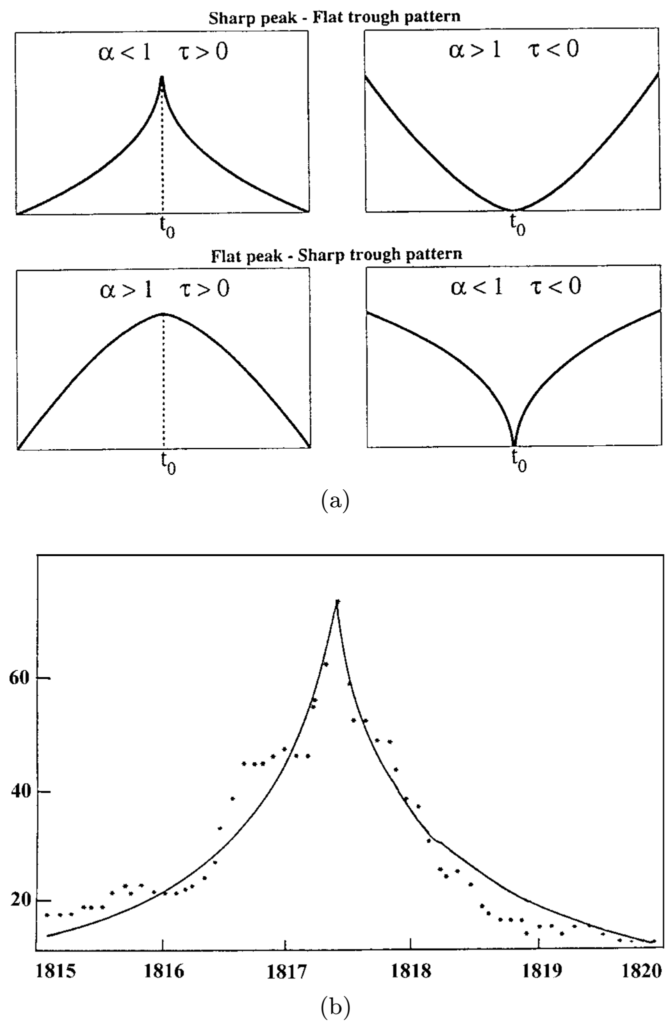


Fig. 3. (a) Peak and trough patterns, (b) Least square fit of the generalized exponential (3) to the price peak in Munich (1815-1819).

2.3.1 Peaks and troughs

Table 1b summarizes the statistical findings for the first set of parameters. The statistical results cannot rule out the null hypothesis that the peaks are symmetric with respect to a vertical line drawn through the turning point. In other words, after a speculative bubble, the prices come back on average approximately to their initial level. Figure 3b provides a typical fit (to the peak price series in Munich from 1815 to 1820) using the parametrization (6).

Table 1c summarizes our results for the values of the parameters α and $|\tau|$ that best fit the peaks and troughs listed in Table 1c. We note the remarkably small dispersion of the determined values of α around

$$\alpha_{peak} = 0.63 \pm 0.03. \qquad (8)$$

Only the Vienna market is somewhat out of range with a larger $\alpha = 0.8$ and thus milder peak structure.

Table 1. (a) Wheat price series analysed in this paper.

Market	Country	Period	Interval [year]	Source
1 Cologne	Germany	1532-1796	265	Ebeling and Irsigler (1976)
2 Grabow	Germany	1785-1870	86	Beiträge zur Statistik (1873)
3 Munich	Germany	1790-1855	66	Seuffert (1857)
4 Paris	France	1521-1698	178	Baulant and Meuvret (1960)
5 Toulouse	France	1486-1913	428	Frêche (1967), Drame *et al.* (1991)
6 Vienna	Austria	1692-1913	222	Pribram (1938)
		Total	1248	years

Table 1. (b) Average values of amplitude, duration and symmetry parameters for peaks and troughs.

		Amplitude > 50%		Amplitude > 100%	
		Peaks	Troughs	Peaks	Troughs
Amplitude	h	2.5 ± 0.2	2.4 ± 0.2	3.2 ± 0.3	3.1 ± 0.3
Duration [month]	d	40 ± 3	42 ± 4	44 ± 5	53 ± 9
Left price/right price	r_p	1.0 ± 0.06	1.1 ± 0.1	1.0 ± 0.1	1.1 ± 0.2
Left durat./right durat.	r_d	1.2 ± 0.2	1.7 ± 0.6	1.4 ± 0.3	1.3 ± 0.4
Left slope/right slope	r_s	1.5 ± 0.3	1.0 ± 0.1	1.0 ± 0.2	1.2 ± 0.4
Number of fluctuations		103	79	44	22
Average interval between fluctuations [year]		8.1	11.5	23.2	49.4

Notes: By and large these data are consistent with peaks lasting about 4 years and which are symmetrical with respect to a vertical line drawn through their summit.

Table 1. (c) Generalized exponential $\exp[-\text{sgn}(\tau)|(t-t_0)/\tau|^{\alpha}]$ adjusted to peaks and troughs of amplitude larger than 50%.

Market	Number fluctuations	α	τ [month]	Goodness of fit
		1) Peaks		
Cologne	13	0.64	27.3	10%
Grabow	5	0.60	23.8	15%
Munich	7	0.63	38.9	11%
Paris	26	0.67	16.9	14%
Toulouse	36	0.62	26.7	13%
Vienna	16	0.80	23.5	10%
Average		0.66	26.2	
		2) Troughs		
Cologne	4	1.14	−17.0	12%
Grabow	5	1.17	−15.4	10%
Munich	7	1.30	−7.7	12%
Paris	16	1.23	−9.8	17%
Toulouse	31	1.05	−17.2	15%
Vienna	16	1.08	−20.0	11%
Average		1.16	−14.5	

Notes: The goodness of fit is defined by the ratio: $e = \sum_i (t_i - o_i)^2 / \sum_i {o_i}^2$ where t_i are the theoretical values and o_i the observations.

Table 1. (d) Estimates for the parameter α for individual price peaks of amplitude larger than 100% on the market at Toulouse.

Year	1498	1508	1516	1529	1539	1546	1563	1573	1580	1595
α	0.453	0.455	0.482	0.509	0.512	0.773	0.784	0.644	0.775	0.500
h	3.1	3.2	2.1	4.0	3.3	4.6	3.3	4.0	2.2	2.1

Year	1614	1631	1644	1694	1710	1713	1720	1812	1817
α	0.750	0.581	0.597	0.495	0.894	0.606	0.613	0.625	0.595
h	2.1	3.6	3.8	2.5	3.7	2.2	3.5	3.5	2.6

Notes: The values of α do not have any definite trend in the course of time. Neither is there any correlation between α and the amplitude of the peak h.

It is also noteworthy that no detectable variation of α or τ can be detected in the course of time. This becomes even more apparent if we list the results for individual peaks (Tab. 1d). Neither is there any correlation between α and the amplitude of the peak h. In sum, the average values of α and τ for peaks versus troughs are consistent with a SP-FT pattern.

2.3.2 The turning point

In the parameterization (6), the turning point t_0 is a singular point due to the absolute value (*i.e.* the expression of $p(t)$ given by (6) is not differentiable at t_0). Is this singularity solely a result of this choice of parametrization or does it correspond to some real economic features? We believe that the latter holds and that the singularity reflects a genuine cooperative behavior of the market not unsimilar to those studied in critical phenomena in Physics.

Indeed, during a major price peak, the spatial correlation length of the markets, quantifying the correlations accross markets in different geographical areas, tends to become very large. In reference [47], it has been shown that during the price boost of 1816-1819 the spatial correlation length jumped to a level about 20 times above its "normal" level. This could be interpreted in two different ways. First one could think that it is the occurrence of an exogenous perturbation that forced prices upward on all markets, thus producing almost simultaneous price jumps throughout the country. Alternatively one may interpret the jump in the correlation length as reflecting a genuine endogenous increase of the strength and range of the interactions between different markets. To distinguish between the two interpretations, the correlation length of meteorological factors was calculated [46] and it was found that there is no significant increase during or even before the occurrence of the peaks. In further support to the second interpretation of an endogenous cause, we note that it has been emphasized by many historians [39] that wheat trade expanded in periods of high prices both in volume and in range. Locally, many self-appointed retailers emerged; regionally, traders were combing the countryside in order to buy every available bushel of wheat for the supply of the cities (even at the expense of the countryside supply); internationally, the government often encouraged (and even subsidized) the importation of additional quantities of wheat from distant markets.

3 Twentieth century commodity markets

3.1 An overview

Whereas in the previous section, it was possible to carry out a systematic analysis with a relatively large data set, the situation is much less favorable for the twentieth century. For most commodities, there have been only a few major peaks during this century. What makes things even worse is that during the first half of the century, the world economy has been disrupted by three major perturbations: World War I, the Great Depression and World War II. For this reason, we restrict our attention to the second half of the century. Our objective in this section is to show by a few examples that the SP-FT pattern also applies to some twentieth commodity markets. A more systematic investigation will be left to a subsequent paper.

In order to give an overview of twentieth century commodity markets, we have listed in Table 2 some of their main characteristics. We note that the number of peaks in the interval considered is too small to give robust estimates of α and τ and the results are thus less reliable that for Table 1c. In order to minimize this difficulty, α and τ have been computed only for the largest peaks with amplitudes larger than 4. It should be emphasized that such data are taken from various sources scattered throughout the literature; this is a field for which a comprehensive handbook of empirical data would be very valuable.

There is a marked contrast between high- and low-volatility commodities (recall that the volatility is the economic term for the coefficient of variation, *i.e.* the ratio of the standard deviation to the mean). The most volatile commodity is sugar, whereas bananas are the least volatile. Between those two products, there is a ratio of 3.4 in terms of coefficients of variation, and of 5.1 in terms of the amplitude of the largest peak. In the following, we examine two cases: sugar and the precious metals, gold and silver.

3.2 Sugar

Figure 4a depicts the shape of the three major peaks that occurred for sugar prices in the interval 1950-1988. We get the following estimates for the parameters α and τ:

1962-1965: $\alpha = 0.97$, $\tau = 11.3$,
1972-1976: $\alpha = 0.63$, $\tau = 7.3$,
1978-1981: $\alpha = 1.03$, $\tau = 9.5$.

Table 2. The response of commodity prices to exogenous pertubations.

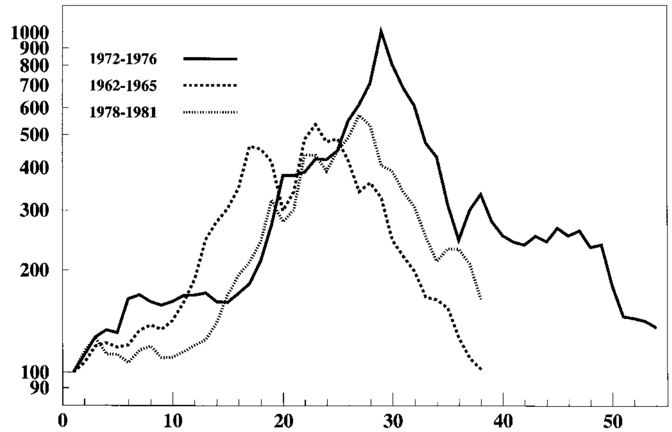
Price elasticity of demand (short-run)	Price elasticity of supply (short-run)	Storage/transportation cost [present]	Commodity	Coefficient of variation 1951-1975	Amplitude of the largest peak 1960-1988	Duration of the largest peak [year]	α	τ month
−0.37	high	30	Bananas	0.18	1.8	0.8		
−0.04		8−25	Sugar	0.61	7.5	4.3	0.88	9.4
−0.10		5.7	Wheat		3.5	5.0		
		1.7	Cocoa	0.29	3.6	7.0		
−0.17	0.06	2.6	Coffee	0.25	4.5	3.4	0.88	42
−0.20	0.15	2.8	Tea	0.26	2.7	4.5	0.90	10
			Copra	0.14	5.9	3.5		
		2.2	Cotton	0.22	2.5	2.4		
			Jute	0.21	3.1	3.8		
−0.30	0.06	0.9	Copper	0.33	2.8	3.0		
			Lead		3.1	8.9		
			Tin	0.20	2.8	10.0		
			Zinc		4.3	5.4	0.72	82
		very low	Gold		8.0	5.4	0.36	174
		low	Silver		6.6	19.0	0.51	41

Notes: Storage and transportation are two closely related operations. Storage costs are per ton-year and include handing costs; they are expressed in percentage of the commodity's average price; since in the case of sugar the very notion of an average price makes little sense, we have indicated a range. In the case of bananas, storage is in practice limited to the one (or one and a half) month taken by their transport and end of ripening; hence the figure given is the relative cost of freight. Let us recall that a high price elasticity of demand ($e = (dD/dp)/(D/p)$) means that the price is fairly independent of the production level, for instance because the demand can be satisfied by shifting to substitutes. Similarly a high elasticity of supply means that production can easily adapt to any demand level; for instance land has a low elasticity of supply. Generally speaking, elasticities are fairly variable not only in the course of time but also from one country to another. Furthermore they cannot "explain" the scale of the peaks since they are themselves derived from an analysis of the price series. Since the prices of silver and gold are highly nonstationary it would be meaningless to compute their volatilities (the result would be strongly time-dependent). The number of the peaks in the interval considered is too small to give "robust" estimates for α and τ; in order to alleviate this difficulty α and τ have been computed only for the largest peaks, *i.e.* for peaks with amplitude larger than 4. Sources: Price elasticities: Huang (1993), Radetzki (1990), Schultz (1938); Storage/transportation costs: Hissenhoven (1923), Mc Nicol (1978), Maillard (1991); Montly prices: Monthly commodity (1985, 1990), Commodity trade and price trends (1988).

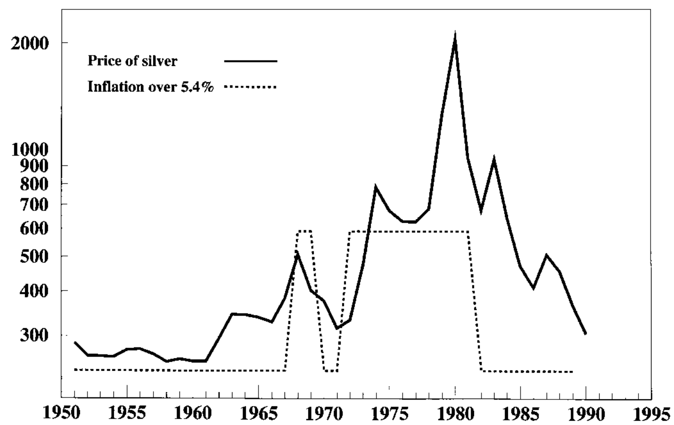
Table 3. Parallel evolution of the prices of paintings and of stocks (1920-1935).

Year	1920	1921	1922	1923	1924	1925	1926	1927	1928	1929
Paintings	100	107	80	147	100	117	142	125	158	125
Dow Jones (industrials)	100	74	94	95	110	145	152	184	273	346
Year	1930	1931	1932	1933	1934	1935				
Paintings	143	283	63	72	100	118				
Dow Jones (industrials)	358	176	81	99	101	135				

Notes: The price index for paintings has been contructed by Buelens and Ginsburgh; the stock prices refer to the yearly highs of the Dow Jones index at the New-York Stock Exchange. Sources: Buelens *et al.* (1993), Farrel (1972).



(a)



(b)

Fig. 4. (a) Price peaks for sugar, (b) annual prices of silver in New York from 1950 to 1990 and inflation rates.

Two of the values of α are close to 1, a situation which was fairly rare for wheat markets in the previous century. We also note that τ is a factor of two or three smaller than before.

3.3 Gold and silver

The case of the silver and gold bubbles which culminated in January 1980 is of special interest for at least three reasons.

1. These peaks were of colossal dimensions both in breadth and in height: the duration was of the order of 20 years and the prices were multiplied by a factor of the order of 20; if the amplitude of the peak shown in Figure 4b appears to be smaller, this is because it is based on annual average prices; in terms of weekly prices, the summit of the peak is as high as 5000 cents/ounce.
2. Here, in contrast to many other cases, the phenomenon which triggered speculation is fairly well-identified; it was the fear of inflation which led many oil magnates to invest in precious metals. This is quantitatively illustrated in Figure 4b; qualitatively it is well-described in a fascinating book by Fay [15].
3. Finally, among all commodities, silver and gold have particularly low (relative) storage/transportation costs. This therefore provides strong evidence against any theory of speculative bubbles which would solely rely on the impact of storage/transportation costs.

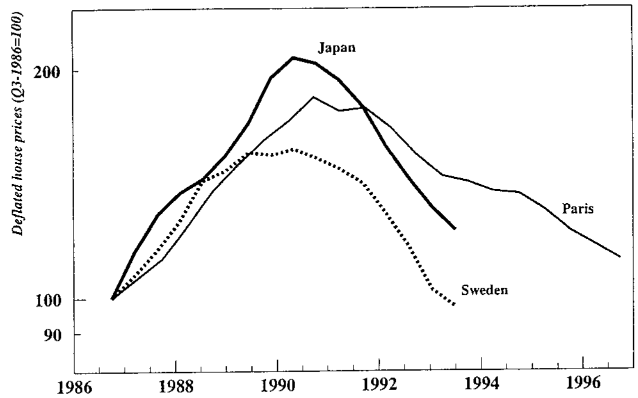


Fig. 5. Real estate price bubbles.

3.4 Price bubbles in other sectors

In this paragraph, let us attempt a generalization. Indeed, it may be illuminating to look at speculative phenomena for which "experimental" conditions are somewhat different. This can help us to separate the crucial variables from those which are specific to a given environment, a prerequisite for a future theory of collective speculative behaviour.

3.4.1 Real estate prices

We consider in Figure 5 the case of real estate prices, a "product" characterized by a very small price elasticity of supply especially in city centers, by long transaction times, and by fairly low storage costs. As a result, the real estate price bubbles turn out to follow a flat peak-flat trough pattern. Indeed, the very small price elasticity of supply is probably at the origin of a limited amplification of the speculative bubble, therefore preventing an acceleration of the price. There are counter examples, for instance in the speculative bubble of Florida real estate market in the twenties (preceeding by a year the Oct. 1929 stock market crash), in which the elasticity of supply was large because new lands were plentiful as previously considered useless lands were made avalaible to buyers [20].

3.4.2 Bubbles in collector's items

Price bubbles in collector's items such as paintings, collectible autos or rare stamps often parallel bubbles in the stock markets or in real estate markets; here we shall restrict ourselves to illustrating this assertion by a few examples.

- Table 3 shows the parallel evolution in the price of paintings as computed by Buelens and Ginsburgh [10] and in the price of stocks on the New-York Stock Exchange. The amplitude of both peaks is of the order of 3 but the turning point in the price of paintings occurred one year after the turning point for stocks. Incidentally, it should be noted that the price boost of the late 1920s did not simply reflect a general increase in the supply of money; in fact, between 1920 and 1929 the total currency mass in the United-States increased by only 4.7% [24].
- In the late 1980s, there has been a speculative bubble in collectible autos. Let us just mention one example: on the London market, the price of a Ferrari 275 GTB4 built between 1966 and 1968 leapt from around \$ 90′000 in 1987 to \$ 1 million in 1989, a multiplication by a factor of 11; by the end of 1991, it had fallen back to \$ 270′000 (Herald Tribune, 28 May 1995).
- In Hong Kong, the prices of rare Chinese stamps have risen steadily since 1970; yet, the trend accelerated at a dizzing pace in the early 1990s. Let us mention two examples: the Chinese 1897 One Dollar stamp has risen from \$ 1′570 in 1970 to \$ 251′000 in 1995, a multiplication by a factor 160; another stamp, the Year of the Ram, had its price multiplied by a factor 430 between 1967 and 1995 (Herald Tribune 28 May 1995).

Note in these examples the unexpectedly large magnitude of the price jump.

These are rather fragmentary indications; unfortunately the data are too sparse and do not allow for an analysis of the shape of the bubbles. Our aim in mentioning them was to emphasize that the phenomenon of speculative bubbles extends beyond the spheres of economics and finance. We think that this observation may be of interest in the perspective of constructing a general theory of speculative behaviour.

3.4.3 Stock market crashes

Large stock market crashes are one of the most dramatic examples of speculative bubbles culminating in peaks preceeded by an acceleration. It has been shown that the largest stock market crashes in this century are outliers: they occur much more frequently that would predict the extrapolation of the historically determined distribution based on the more numerous smaller price variations, even when taking into account the significant deviation from the Gaussian law [28]. The apparent disappearance of the SP-FT structure in the present century for the "common" peaks is paralleled by the very strong sharp peak pattern accompanying the largest crashes. This result suggests that large crashes result from amplification processes that have not been washed out by the large liquidity of the modern markets. These amplification processes (of speculative nature) could be similar to those at the origin of the sharp peak structure on commodity prices observed in the past centuries.

To pursue the analogy, we note that the exponents α reported in Table 1c are remarkably close to those obtained for the largest crashes of this century. When fitting the price $p(t)$ directly to a power law (7) (taking into account additional log-periodic corrections [52]), we find an exponent $\alpha \approx 0.3$, that is half the average value reported in Table 1c. In contrast, if we fit the logarithm of the price to a power law (with again log-periodic corrections [53]), we find an exponent $\alpha \approx 0.6$ for the Oct. 1929, the Oct. 1987, the May 1962 "slow crash" [27], the Hong-Kong Oct. 1997 crash [26] and the black monday of October 31, 1997 on the US equity market [59]. This value $\alpha \approx 0.6$ is remarkably similar to those found for the sharp peaks of commodities in the previous centuries. Due to increased liquidity and efficiency in modern markets, the sharp peak patterns have essentially disappeared except for the most dramatic crashes for which the precursory patterns develop over so long time scales (8 years [53]) that they have not yet been adequately arbitraged away by the market. The exponent $\alpha \approx 0.6$ might be the signature of the universality class of speculative/imitation behavior close to the critical point (the turning point of the sharp peak). However, for crashes, the symetric structure of the peaks disappear.

4 Toward a theory

4.1 Linear stochastic multiplicative models

There is a large literature on the use of auto-regressive models to model economic times series. As already pointed out, the econometric nonlinear autoregressive models with conditional heteroskedasticity (ARCH: autoregressive-conditionally-heteroscedastic) [17] and their generalizations [6] keep the volatility (standard deviation of price variations) as the main descriptor and allow for the fact that the variance (or volatility) is itself a stochastic variable. In the simplest version of ARCH(1), we have the following stochastic difference equation for the log-returns:

$$R_{t+1} = \sqrt{b + aR_t^2}\; Z_t, \tag{9}$$

where Z_t is a Gaussian random variable of zero mean and unit variance. This process (9) describes a persistence and thus clustering of volatilities R_t^2. Indeed, the factor $(b + aR_t^2)^{1/2}$ ensures that the amplitude of the motion R_{t+1} is controlled by the past realization of the amplitude R_t^2. Now, calling $X_t \equiv \langle R_t^2 \rangle$, where the average is carried out over the realization of Z_t, we see that (9) is equivalent to

$$X_{t+1} = aX_t + b. \tag{10}$$

We now allow the coefficient a and b to depend on time (independently from Z_t) leading to an equation with stochastic coefficient. This representation (10) provides useful results under the following conditions. Let us assume that

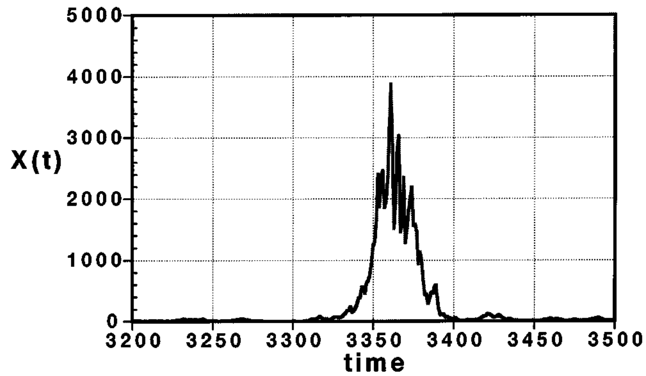
b is always positive and it may or may not fluctuate. Its presence ensures that X does not shrink to zero asymptotically, even if the a's are less than one. We then imagine that the multiplicative factors a are drawn from some distribution such that on average the rate of growth is negative, thus preventing the explosion of the process, but with intermittent realizations of a larger than one. Intuitively, we can think that the realization of a reflects the meteorological factor, with $a < 1$ corresponding to favorable conditions while $a > 1$ corresponds to bad weather with a negative impact on wheat production. b can reflect a basic contribution of the price, like the minimum wage and price of production that may fluctuate but is strictly positive. If the meteorological conditions are always favorable, the realizations of a are always less than one and X converges asymptotically to the fundamental price imposed by the price of production. In contrast, in the presence of intermittent adverse factors, a may become larger than one one for several time steps in a row, leading to a transient exponential growth.

This process (10) has a long history. See [51] for a review with applications to population dynamics with external sources, epidemics, finance and insurance applications with relation to ARCH process, immigration and investment portfolios, congestions on the internet, the statistical physics of directed polymers in random media, autocatalytic chemical reactions, *etc.* At first sight, it seems that the linear model (10) is so simple that it does not deserve a careful theoretical investigation. However it turns out that this is not the case: see, for example, a rather complicated mathematical analysis of the problem in [30]. It turns out that model (10) exhibits an unusual type of intermittency with a power law probability distribution of the variable X_t, for a large range of distributions for a and b. The non-trivial properties of this simple model (10) come from the competition between the multiplicative and additive terms [54].

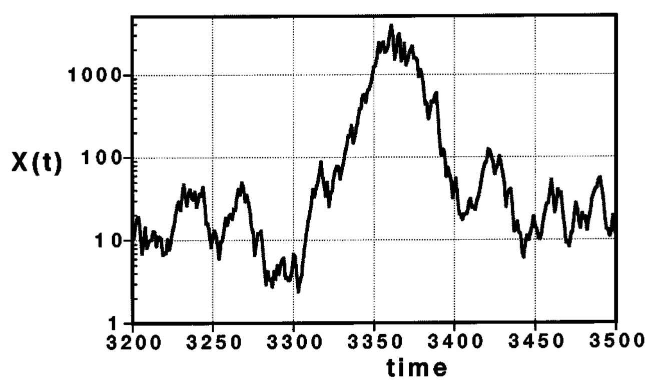
The formal solution of (10) for $t \geq 1$ can be obtained explicitly

$$X_t = \left(\prod_{l=0}^{N-1} a(l)\right) X_0 + \sum_{l=0}^{N-1} b(l) \prod_{m=l+1}^{N-1} a(m), \qquad (11)$$

where, to deal with $l = N-1$, we define $\prod_{m=N}^{N-1} a(m) \equiv 1$. Because of the successive multiplicative operations on a in the iteration of X_t, it is clear that the behavior is locally exponentially increasing or decreasing. The Figure 6a shows a times series generated for a and b uniformly distributed in the intervals $0.48 \leq a \leq 1.48$ and $0 \leq b \leq 1$. In this case $\langle \log a \rangle = -0.06747$ and $\langle a \rangle = 0.98$. The Figure 6b shows the same realization with a logarithmic scale in X, clearly demonstrating the tent-like peak structure. Thus, we conclude that stochastic multiplicative processes account for the intermittent production of sharp peaks, but the upward concavity quantified by (6, 7) with $\alpha = 0.6$ is not captured. In contrast to our empirical finding, this class of model predicts an exponent $\alpha = 1$. This result is very instructive: stochastic multiplicative Kesten processes account for a lot of observations, such as distributions of increments with fat (power law) tails, self-affinity of the time series, multifractality, volatility bursts, but they are not capable of representing the acceleration preceeding large peaks that are characterized by an exponent α significantly less than one.



(a)



(b)

Fig. 6. (a) Times series $X(t)$ generated for by equation (10) for a and b uniformly distributed in the intervals $0.48 \leq a \leq 1.48$ and $0 \leq b \leq 1$. In this case $\langle \log a \rangle = -0.06747$ and $\langle a \rangle = 0.98$. For these parameters, $\langle X_t \rangle = \frac{\langle b \rangle}{1-\langle a \rangle} = 25$. Most of the time, X_t is significantly less than its average, while rare intermittent bursts propel it to very large values, (b) Same as Figure 6a but with a logarithmic scale for $X(t)$ showing a tent-like structure of the peaks.

4.2 Nonlinear models

We propose a nonlinear amplification phenomenon. To illustrate the idea, consider as a toy model the following equation describing the price increments δx

$$\frac{d\delta x}{dt} = \kappa (\delta x)^{\beta}, \qquad (12)$$

with $\beta > 1$, whose solution reads

$$\delta x(t) \sim |t_0 - t|^{1/(\beta - 1)}. \qquad (13)$$

Its time integral provides

$$x(t) \sim |t_0 - t|^{\alpha} \qquad (14)$$

with

$$\alpha = \frac{\beta - 2}{\beta - 1}, \tag{15}$$

which recovers (7). The observed value $\alpha \approx 0.6$ translates into $\beta \approx 3.5$. The intuitive interpretation of a value $\beta > 1$ is to represent multi-body interactions between several players. The expression (12) is then similar to a "mean-field" equation representing the average behavior of a representative agent interacting with an effective number β of other agents. This interaction, embodying the processes of imitation and herding, is responsible for the observed acceleration preceeding large peaks.

While suggestive, this model remains very schematic and would need to be developed to account for the heterogeneity of the agents and the stochastic factors entering the market. This is left for a future work.

5 Conclusion

We have identified empirically a characteristic sharp peak-flat trough pattern in a large set of commodity prices. These patterns provide a demonstration that those markets, that exhibit them, have not yet reached a fully efficient regime and these patterns constitute recognizable signature of impending correlated price series. We have shown that similar behaviors occur in a large variety of markets, all the more so, the less liquid is the market. Using simple models, we have shown that nonlinear amplification processes must be invoked to account for the observed acceleration. Mechanistically, the nonlinear behavior embodies multi-body interactions leading to imitation and herding.

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